PROCESS DESIGN AND CONTROL

On the Use of Net Present Value in Investment Capacity Planning Models

Miguel Bagajewicz†

University of Oklahoma, 100 E. Boyd T-335, Norman, Oklahoma 73019

Net present value has been widely used in several investment planning models. This article examines the validity of this objective function, discusses its merits, and also provides some simple alternatives.

Introduction

Net present value (NPV) is being used in many investment planning problems. The measure was used by almost all researchers, including me. For example, in investment planning our work on financial risk[1,2] uses NPV as a profitability measure. All the previous work on this problem[3−6] also used this measure. The NPV measure was for example used in the oil and gas drilling and production problem,[7−12] although in this problem sometimes cost was minimized given fixed demand profiles: Ortiz-Gomez et al.[13] used minimum cost and Kosmidis et al.[14] used pure profit without subtracting investment. NPV is also used in petroleum exploration.[15] Conceptual development work, such as the one by Liu and Sahinidis,[16] Iyer and Grossmann,[17] Bok et al.,[18] and Cheng et al.[19] also used NPV as the profitability measure of choice.

In this article, we concentrate on the use of NPV in investment planning models used in the process systems literature. We first review the nomenclature, and then we discuss the merits of using NPV and offer some alternatives. We consider projects of the same length.

Profitability Measures

We now concentrate on the different profitability measures used in investment valuation. We start with net present value (NPV), which is the object of this article, and we compare it to different forms of return of investment. Net present value (NPV) is defined as follows,

$$\mathrm{NPV} = \sum_{i=1}^{N} d_{i,0}\mathrm{CF}_i - I \tag{1}$$

where d_i are the discount factors, CF_i are the after taxes cash flow of period, and I the capital investment.[20−22] In turn the discount factor $d_{i,0}$ is given by

$$d_{i,0} = \frac{1}{(1+r)^i} \tag{2}$$

where r is the minimum expected rate of return, also called the “opportunity cost of capital”;[22] that is, all future cash flows are referred to the present. Engineering books like the popular Biegler et al.,[23] Peters et al.,[24] or Seider et al.[25] are a bit fuzzy about the choice of this rate.

In engineering, the evaluation of projects is also performed using a variety of measures besides net present value. Among them, the internal rate of return (IRR), the return of investment (ROI), and the payback period or payout time.[23−25] All these measures are aimed at reducing the complex process of cash flow that takes place in different periods of time in the future to one single number. In this article I concentrate on discussing the merits of NPV in view of what ROI may offer.

The internal rate of return (IRR) is given by the solution to

$$\sum_{i=1}^{N} \frac{\mathrm{CF}_i}{(1+\mathrm{IRR})^i} = I \tag{3}$$

which can be considered as the rate of return that makes the NPV equal to zero. IRR has its appeal in practice because of the tendency to look at investments in terms of percentage return of the capital invested. The number, however, does not take into account the time value of money. Thus, an alternative way of estimating this number is through the modified IRR (MIIR),[22]

$$\frac{\sum_{i=1}^{N} d_{i,N}\mathrm{CF}_i}{(1+\mathrm{MIRR})^N} = I \tag{4}$$

where the discount factor $d_{i,N}$ is given by

$$d_{i,N} = (1+r)^{N-i} \tag{5}$$

This form, which makes more sense, assumes that one invests the proceeds CF_i at some rate (different from MIRR) to collect all of them at once in year N.

IRR or MIRR suffer from several drawbacks that are known:[20,22]

• It does not distinguish between lending and borrowing, which is an issue that does not affect investment planning models of the type considered here (lending is not usually included, although borrowing is sometimes considered).

• It can have multiple solutions. In turn, return on investment (ROI) is defined in its most simplified form as follows,

$$\mathrm{ROI} = \frac{\left(\sum_{i=1}^{N} \mathrm{CF}_i\right)/N}{I} \tag{6}$$

where N is the number of years of the project and an average value of the after tax revenues is used. By dividing by the investment, one can obtain the rate at which the investment is recovered. Until engineers took a serious look at investment planning problems with varying demands and prices through time, or uncertainties, they were taught to assume that all cash flows from a project are equal, which simplified the expression. The expression given in (6) is therefore a direct extension of this criterion, aimed at obtaining some kind of “averaged” return. One can also define an ROI for each year, which can give a distribution of values as follows:

† E-mail: bagajewicz@ou.edu.

10.1021/ie071008l CCC: $40.75
Published on Web 11/07/2008

$$\mathrm{ROI}^{(i)} = \frac{\mathrm{CF}_i}{I} \tag{7}$$

It is easy to see that ROI is the average of the yearly values $\mathrm{ROI}^{(i)}$. One can also use the modified ROI (MROI) that takes into account the time value of money, referring it to the present,

$$\mathrm{MROI}^0 = \frac{\left(\sum_{i=1}^{N} d_{i,0}\mathrm{CF}_i\right)/N}{I} \tag{8}$$

NPV has been chosen by the engineering community as the preferred objective function over IRR, aside from its inherent appealing value, because it is linear, and since most models have linear constraints, the use of an MILP structure allows global optimality and the use of existing codes like CPLEX. In addition, ROI has not been favored because it is not as highly valued as a tool as NPV in finance circles,[20−22] it does not take into account the time value of money (in its simpler forms), and it is also nonlinear. The aforementioned literature on finances coincides that the return of investments when the net income is used in its accounting form is seriously misleading. However, Emery[20] points out that when the net proceeds of the investment are counted in a cash and not accrual form, then ROI is a valid profitability measure.

Effect of Uncertainty

Expected net present value is usually defined in its discrete (scenario-based) form as

$$\mathrm{ENPV} = \sum_{s=1}^{S} p_s \mathrm{NPV}_s \tag{9}$$

where p_s is the probability of scenario s, S the total number of scenarios, and NPV_s the net present value of scenario s. Note that, in principle, the capital investment I stays the same for all scenarios.

Gupta and Maranas[26] suggested using real-options-based valuation (ROV) instead of expected net present value (ENPV). Real-options-based valuation uses a risk-free rate of return whereas NPV uses the expected one.[22] In addition, ROV uses risk-neutral probabilities whereas ENPV uses real probabilities. Despite this better handling of uncertainty, the basic structure of computing profitability by discounting future cash revenues and subtracting cost is at the core of both ideas. We do not focus on this aspect of planning in this article, but we believe several of the conclusions for NPV hold for ENPV as well.

Comparison of NPV and ROI

Academics and practitioners in the finance side point out that NPV is a better profitability measure than IRR.[22] They also dismiss ROI without giving too many counter examples. Brealey and Myers[22] reason that the version of ROI that is commonly used is based on book value and not on real cash flows. However, the engineering literature presents it as a valid profitability measure. We now explore the conditions under which they point to the same optimum.

This widely used measure of profitability may lead to situations where NPV is the maximum, while the return of investment is not maximized. We explore the consequences now.

Consider two solutions of the NPV maximization problem, NPV_1 being the optimal one and NPV_2 another one that is positive. Assume also that the cash flows are different, then

$$\Delta\mathrm{NPV} = \mathrm{NPV}_1 - \mathrm{NPV}_2 = \sum_{i=1}^{N} d_{i,0}(\mathrm{CF}_{i,1} - \mathrm{CF}_{i,2}) - (I_1 - I_2) \geq 0 \tag{10}$$

In turn,

$$\mathrm{MROI}_1^0 - \mathrm{MROI}_2^0 = \frac{\left(\sum_{i=1}^{N} d_{i,0}\mathrm{CF}_{i,1}\right)\Big/N}{I_1} - \frac{\left(\sum_{i=1}^{N} d_{i,0}\mathrm{CF}_{i,2}\right)\Big/N}{I_2} \tag{11}$$

Assume also that $I_1 = I_2 + \Delta I$. Then

$$\sum_{i=1}^{N} d_{i,0}(\mathrm{CF}_{i,1} - \mathrm{CF}_{i,2}) = \Delta\mathrm{NPV} + \Delta I \tag{12}$$

and therefore

$$\mathrm{MROI}_1^0 - \mathrm{MROI}_2^0 = \frac{\Delta\mathrm{NPV} + \Delta I\left(1 - \left(\sum_{i=1}^{N} d_{i,0}\mathrm{CF}_{i,2}\right)\Big/I_2\right)}{N(I_2 + \Delta I)} \tag{13}$$

Because NPV_1 is optimal, then $\Delta\mathrm{NPV} > 0$. Thus, under the condition,

$$\Delta\mathrm{NPV} < \Delta I\left(1 - \left[\sum_{i=1}^{N} d_{i,0}\mathrm{CF}_{i,2}\right]\Big/I_2\right) \tag{14}$$

we would obtain $\mathrm{MROI}_1^0 - \mathrm{MROI}_2^0 < 0$. Notice that this requires $\Delta I > 0$ (the term in parentheses is positive because $\mathrm{NPV}_2 > 0$); that is, the second project, which by definition has positive cash flows, has to have a smaller investment. We will see this feature showing up in the example. Thus, when $\Delta I < 0$, both measures point to the correct optimum.

Thus, under the condition given by (14), one can arrive at a different conclusion regarding profitability. In other words, *a solution optimal from the point of view of maximizing NPV can be less than optimal from the point of view of return of investment and vice versa.*

If one wants to use the nondiscounted definition of ROI given by (6), then we have

$$\mathrm{ROI}_1 - \mathrm{ROI}_2 = \frac{1}{N}\left[\frac{\left(\sum_{i=1}^{N}\mathrm{CF}_{i,1}\right)}{I_2 + \Delta I} - \frac{\left(\sum_{i=1}^{N}\mathrm{CF}_{i,2}\right)}{I_2}\right]$$

$$= \frac{\vartheta_2\left[\Delta\mathrm{NPV} + \Delta I\left(1 - \frac{\left(\sum_{i=1}^{N} d_{i,0}\mathrm{CF}_{i,2}\right)}{I_2}\right)\right]}{N(I_2 + \Delta I)} \tag{15}$$

where $\vartheta_2 = (\sum_{i=1}^{N}\mathrm{CF}_{i,2})/(\sum_{i=1}^{N} d_i\mathrm{CF}_{i,2})$. Thus, the same conclusions hold. We notice that in both cases, when $\Delta I = 0$, we have

$$\mathrm{MROI}_1^0 - \mathrm{MROI}_2^0 = \mathrm{ROI}_1 - \mathrm{ROI}_2 = \frac{\Delta\mathrm{NPV}}{N(I_2 + \Delta I)} \tag{16}$$

In other words the two measures are consistent (both point to the same optimum) for the same investments. But this is hardly the case in planning models because different feasible solutions have different investments in classical investment planning models.

Alternative Formulations

Assume there are N_s scenarios and let x be the vector of first stage (design) decisions and y_s the vector for second stage (recourse) decisions for scenario s. Then, a compact representation of the planning problem is:

$$\begin{aligned} &P_1 = \underset{x,y_s}{\mathrm{Max}} \sum_s p_s \mathrm{NPV}(x, I, y_s) \\ &\text{s.t.} \\ &g(x, y_s) \le 0 \quad \forall\, s \\ &I(x) \le I_{\max} \end{aligned} \tag{17}$$

where $g(x,y_s)$ represents the set of restrictions. We have also added a specific constraint limiting the capital that can be invested. If one wants to use MROI as a profitability measure, then the problem would have the following nonlinear form:

$$\begin{aligned} &P_2 = \underset{x,y_s}{\mathrm{Max}} \sum_s p_s \mathrm{MROI}^0(x, I, y_s) \\ &\text{s.t.} \\ &g(x, y_s) \le 0 \quad \forall s \\ &I(x) \le I_{\max} \end{aligned} \tag{18}$$

Both problems may give different answers and, very likely, different investment levels.

We now consider for simplicity the deterministic version of the two problems (one scenario). Consider the case in which $\Delta I > 0$; that is, problem P_1 has a larger investment associated with the maximum ENPV. This means that the excess capital allowed a larger return in time, but at a slower pace. Assume we solve problem P_1 and obtain a set of cash flows CF_i* and capital I* with a corresponding net present value given by NPV*. Assume also that problem P_2 renders a set CF_i^+ and a capital investment $I^+ < I$*, with a value of MROI^+. Then we can write

$$\begin{aligned} \mathrm{NPV}^* &= \sum_{i=1}^{N} d_i \mathrm{CF}_i^* - I^* \\ &= \sum_{i=1}^{N} d_{i,0} \mathrm{CF}_i^+ + \sum_{i=1}^{N} d_{i,0} \times (\mathrm{CF}_i^* - \mathrm{CF}_i^+) - I^+ - (I^* - I^+) \\ &= (N\,\mathrm{MROI}^+ - 1)I^+ + (N\,\mathrm{MROI}' - 1)(I^* - I^+) \end{aligned} \tag{19}$$

where

$$\mathrm{MROI}' = \frac{\sum_{i=1}^{N} d_i(\mathrm{CF}_i^* - \mathrm{CF}_i^+)}{N(I^* - I^+)} \tag{20}$$

which corresponds to the rate of return of the difference in capital of the two solutions. Because the two solutions have different capital investment, then we already know that MROI′ < MROI^+. In other words, the excess capital ($I^* - I^+$) is being invested with smaller return.

Conceivably, one might have another project where such capital could be invested. If such project has an MROI that is larger than MROI′, it would be foolish not to invest in such project. The problem is that one does not know what is the possible "extra" investment before solving both problems. To overcome this, I propose some alternative procedures:

Alternative 1. Solve P_2, obtain I^+, and then solve the following problem:

$$\begin{aligned} &P_3 = \underset{x,y_s}{\mathrm{Max}} \sum_s p_s \mathrm{NPV}(x, I, y_s) \\ &\text{s.t.} \\ &g(x, y_s) \le 0 \quad \forall\, s \\ &I(x) \le I_{\max} \\ &\frac{\sum_{i=1}^{N} d_i \mathrm{CF}_{i,s} - N\,\mathrm{MROI}^+}{N[I(x) - I^+]} \ge \mathrm{MROI}_{\mathrm{ref}} \end{aligned} \tag{21}$$

The new constraint basically states that the extra capital should only be invested if the extra cash generated has a rate of return that is larger than some reference rate $\mathrm{MROI}_{\mathrm{ref}}$. Fortunately, because $\mathrm{MROI}_{\mathrm{ref}}$ is a constant, the new added constraint can be rewritten in a form that makes it linear.

Alternative 2. Always solve P_1 in such a way that all the available capital is invested. To do so, one can try to add the planning of other projects, or simply add the possibility of investing the capital in the stock market in securities. That is, solve

$$\begin{aligned} &P_4 = \underset{x_k,y_s}{\mathrm{Max}} \sum_{k \in K} \sum_{s_k} p_{s_k} \mathrm{NPV}_k(x_k, I_k, y_{k,s}) + \\ &\qquad N\,\mathrm{MROI}_{\mathrm{ref}}\Big[I_{\max} - \sum_k I_k(x)\Big] \\ &\text{s.t.} \\ &g(x_k, y_{k,s}) \le 0 \quad \forall\, s_k \\ &\sum_k I_k(x) \le I_{\max} \end{aligned} \tag{22}$$

In this problem, we consider a set of projects K, each one with its own set of scenarios s_k. The extra capital not invested in these projects is assumed to be invested in ventures outside the company (securities, etc.), which have a rate given by $\mathrm{MROI}_{\mathrm{ref}}$.

Alternative 3. Solve P_1 for different values of $I_{\max}$, calculate the MROI for each solution, and make a judgment. This is the alternative we prefer for its simplicity.

Example

We consider an investment planning problem recently investigated.[27] This investment planning problem studies the selection of technologies and the markets to supply natural gas to stranded locations. These locations have small populations and are scattered throughout a region. The case chosen was Bolivia. The study considers the installation of pipelines between producing centers and distribution centers. The model allows choosing other transportation modes like compressed natural gas or liquefied natural gas transported on trucks. Finally, the model chooses the most attractive markets. Investment costs include liquefaction and compression plants, pipelines, and all the associated equipment. It also takes into account the number of trucks that need to be purchased. The model is stochastic and risk analysis was performed. We omit the details, which can be consulted in the original work.

We concentrate on the results. The model was run with different values of maximum capital. Table 1 summarizes the

Table 1. Profitability Comparison for Solutions Obtained by Vargas[27]

$I_{\max}$ (MM\$)	I (MM\$)	NPV	MROI (%)	MROI′ (%)
2.00	1.99	3.08	12.7	
2.50	2.50	3.80	12.6	12.1
3.00	3.00	4.26	12.1	10.8
4.00	4.00	4.56	10.7	8.7
6.00	4.57	4.60	10	7.9

results. When I_{max} is sufficiently large, the model maximizes NPV and chooses to invest no more than 4.57 MM$. However, if I_{max} is limited to values smaller than 4.57 MM$, one finds that it is always invested completely. For these lower capital investment solutions, however, MROI is higher. The last column shows the profitability of the extra capital, as compared to the solution with highest MROI. It is very clear that one would invest the extra capital in some other ventures if the profitability is higher than MROI′. For example, if there is another investment that has a profitability of say, 10.8, then any capital above 3.00 needs to be invested in the second project.

Conclusions

In this article, we pointed out that when using net present value in planning models, one needs to make sure that capital is utilized at the maximum profitability possible. This is not always achieved running models that maximize net present value. Additional considerations and procedures are needed. At this point, we conclude that these simple alternative solutions are not enough and that what is needed is to consider the whole portfolio of investments for the company over a certain horizon. Thus, we can say that several planning models that have been published need revisiting. Some of this need was hinted at through an example by Biegler et al.,[23] where three investments are compared under a variety of options. We leave this for future work.

Literature Cited

(1) Barbaro, A. F.; Bagajewicz., M. Managing Financial Risk in Planning under Uncertainty. *AIChE J.* **2004**, *50* (5), 963.

(2) Barbaro, A. F.; Bagajewicz., M. Use of Inventory and Option Contracts to Hedge Financial Risk in Planning under Uncertainty. *AIChE J.* **2004**, *50* (5), 990.

(3) Sahinidis, N. V.; Grossmann, I. E.; Fornari, R. E.; Chathrathi, M. Optimization Model for Long Range Planning in the Chemical Industry. *Comput. Chem. Eng.* **1989**, *13*, 1049.

(4) Liu, M. L.; Sahinidis, N. V. Optimization in Process Planning under Uncertainty. *Ind. Eng. Chem. Res.* **1996**, *35*, 4154.

(5) Ahmed, S.; Sahinidis, N. V. Robust Process Planning Under Uncertainty. *Ind. Eng. Chem. Res.* **1998**, *37*, 1883.

(6) Aseeri, A.; Bagajewicz, M. New Measures and Procedures to Manage Financial Risk with Applications to the Planning of Gas Commercialization in Asia. *Comput. Chem. Eng.* **2004**, *28*, 12.

(7) Iyer, R. R.; Grossmann, I. E.; Vasantharajan, S.; Cullick, A. S. Optimal Planning and Scheduling of Offshore Oil Field Infrastructure Investment and Operations. *Ind. Eng. Chem. Res.* **1998**, *37*, 1380.

(8) Van den Heever, S.; Grossmann, I. E.; Vasantharajan, S.; Edwards, K. Integrating Complex Economic Objectives with the Design and Planning of Offshore Oilfield Infrastructures. *Comput. Chem. Eng.* **2000**, *24*, 1049.

(9) Van den Heever, S.; Grossmann, I. E. An Iterative Aggregation/Disaggregation Approach for the Solution of the Mixed Integer Nonlinear Oilfield Infrastructure Planning Model. *Ind. Eng. Chem. Res.* **2000**, *39*, 1955.

(10) Van den Heever, S.; Grossmann, I. E.; Vasantharajan, S.; Edwards, K. A Lagrangean Decomposition Heuristic for the Design and Planning of Offshore Field Infrastructures with Complex Economic Objectives. *Ind. Eng. Chem. Res.* **2001**, *40*, 2857.

(11) Goel, V.; Grossmann, I. E. A Stochastic Programming Approach to Planning of Offshore Gas Field Developments under Uncertainty in Reserves. *Comput. Chem. Eng.* **2004**, *28*, 1409.

(12) Aseeri, A.; Gorman, P.; Bagajewicz, M. Financial Risk Management in Offshore Oil Infrastructure Planning and Scheduling. *Ind. Eng. Chem. Res.* **2004**, *43* (12), 3063.

(13) Ortiz-Gomez, A.; Rico Ramirez, V.; Hernandez-Castro, S. Mixed-integer Multiperiod Model for the Planning of Oilfield Production. *Comput. Chem. Eng.* **2002**, *26*, 703.

(14) Kosmidis, V. D.; Perkins, J. D.; Pistikopoulos, E. N. Optimization of Well Oil Rate Allocations in Petroleum Fields. *Ind. Eng. Chem. Res.* **2004**, *43*, 3513.

(15) Chungchaoren, E.; Fuller, J. D. Risk and Profitability Assessment of Hydrocarbon Exploration Plans. In *Proceedings of the International Conference on Advances in Petrochemicals and Polymers in the New Millennium*, 2003.

(16) Liu, M. L.; Sahinidis, N. V. Bridging the Gap between Heuristics and Optimization: Capacity Expansion Case. *AIChE J.* **1997**, *43*, 2289.

(17) Iyer, R. R.; Grossmann, I. E. A Bilevel Decomposition Algorithm for Long-range Planning of Process Networks. *Ind. Eng. Chem. Res.* **1998**, *37*, 474.

(18) Bok, J.; Lee, H.; Park, S. Robust Investment model for Long-Range Capacity Expansion of Chemical Processing Networks under Uncertain Demand Forecast Scenarios. *Comput. Chem. Eng.* **1998**, *22*, 1037.

(19) Cheng, L.; Subrahmanian, E.; Westerberg, A. W. Design and Planning under Uncertainty: Issues on Problem Formulation and Solution. *Comput. Chem. Eng.* **2003**, *27*, 781.

(20) Emery, G. W. *Corporate Finance*; Addison Wesley: New York, 1998.

(21) Keown, A. J.; Martin, J. D.; Petty, J. W.; Scott, D. F., Jr. *Financial Management: Principles and Applications*; Prentice Hall: Englewood Cliffs, NJ, 2002.

(22) Brealey, R. A.; Myers, S. C. *Principles of Corporate Finance*; McGraw Hill: New York, 2003.

(23) Biegler, L. T.; Grossmann, I. E.; Westerberg, A. W. *Systematic Methods of Chemical Process Design*; Prentice Hall: Englewood Cliffs, NJ, 1997.

(24) Peters, M. K.; Timmerhaus, D.; West, R. E. *Plant Design and Economics for Chemical Engineers*; McGraw Hill: New York, 2002.

(25) Seider, W. D.; Seader, J. D.; Lewin, D. R. *Product and Process Design Principles, Synthesis, Analysis and Evaluation*; Wiley: New York, 2004.

(26) Gupta, A.; Maranas, C. D. Real Options-based Planning Strategies under Uncertainty. *Ind. Eng. Chem. Res.* **2004**, *43*, 3870.

(27) Vargas, W. Case Study: Natural Gas Commercialization in Stranded Markets. M.Sc. Thesis, University of Oklahoma, Norman, OK, 2005.

Received for review July 24, 2007
Revised manuscript received August 13, 2008
Accepted September 5, 2008

IE071008L